INDIAN MARITIME UNIVERSITY
(A Central University, Govt.of India)

May/June 2015 End Semester Examinations

SEMESTER – II, B.TECH (MARINE ENGINEERING)

MATHEMATICS II (T 2202 / T 1202)

Date:08.06.2015 **Max.Marks:100**
Time:-3 Hrs **Pass Marks:50**

PART – A **(3 x10 = 30 Marks)**
(Compulsory Questions)

1. a) Find the Fourier coefficients a_1 and b_1 for the function

$$f(t) = \begin{cases} \sin t, 0 \le t \le \pi \\ 0, \pi \le t \le 2\pi \end{cases}$$

b) Use unit step functions to evaluate the Laplace transform of the following function: $f(t) = \begin{cases} t, 0 < t < 2 \\ 2, t > 2 \end{cases}$

c) Solve the differential equation $\dfrac{dy}{dx} = e^{2x-3y} + 4x^2 e^{-3y}$.

d) Evaluate $L^{-1}\left\{\dfrac{1}{2}\log\dfrac{s^2+b^2}{s^2+a^2}\right\}$.

e) Calculate the PI for the differential equation $(D^2-16)u = 2e^{4x}$.

f) Find only the integrating factor for the non-exact equation

$$(x^2y^3 - y)dx + (x^3y^2 + x)dy = 0.$$

g) In a Binomial distribution the mean is 4 and the variance is 8/3. Find the mode of the distribution.

h) Find the value of k if $f(x) = \begin{cases} kx^2, 0 < x < 3 \\ 0, otherwise \end{cases}$, is a probability density function.

i) If a random variable has a Poisson distribution such that P(1)=P(2), find (i) the mean of the distribution and (ii) P(4).

j) Find the orthogonal trajectory to the family of curves $y = ax^2$.

PART – B **(5 x14 = 70 Marks)**

(Answer any five of the following)

2. a) Find the half range cosine series for the function $f(t)=\begin{cases} t, 0<t<2 \\ 4-t, 2<t<4 \end{cases}$

   b) Find the Fourier series for the function $f(x)=\begin{cases} -1, -\pi<x<-\pi/2 \\ 0, -\pi/2<x<\pi/2 \\ 1, \pi/2<x<\pi \end{cases}$

(7+7)

3. a) Use Convolution theorem to calculate the inverse Laplace transform $L^{-1}\left[\dfrac{1}{(s+2)^2(s-2)}\right]$

   b) Solve the initial value problem $\dfrac{d^2x}{dt^2}+2\dfrac{dx}{dt}+5x=e^{-t}\sin t$, $x(0)=0, x'(0)=1$

(6+8)

4. a) Solve the equation $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$

   b) Find the complete solution (CS) for the differential equation $(D^2+4D+3)y(x)=e^{-x}\sin x+xe^{3x}$

(6+8)

5. a) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+9y=\dfrac{e^{3x}}{x^2}$

   b) When a resistance R ohms is connected in series with an inductance L henries with an e.m.f. of E volts, the current i amperes at time t is given by $L\dfrac{di}{dt}+Ri=E$. Use method of integrating factor to find $i(t)$ with $E=10\sin t$, while the initial condition is $i(t)=0$ at $t=0$

(7+7)

6. a) A problem in mathematics is given to three students whose chances of solving the problem are 1/2, 1/3, and 1/4. What is the probability that the problem is solved

   b) There are three bags; the first containing 1 white, 2 red and 3 green balls; the second containing 2 white, 3 red and 1 green balls and the third bag containing 3 white, 1 red and 2 green balls. Two balls are drawn from a bag chosen at random. They are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag and the first bag.

(6+8)

7. a) Show that the Poisson's distribution is a limiting case of binomial distribution When n is very large and p is very small in such a way that $np \to \lambda$.

b) Out of 800 families with five children each, how many families would be expected to have

i) three boys and two girls (ii) two boys and three girls

iii) at the most two girls, under the assumption that the probabilities for boys and girls are equal.

(6+8)

8. a) Derive the recurring relation for the moments of binomial distribution

$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$ where symbols have their usual meanings.

b) Use method of undetermined coefficients to solve the differential equation

$$\frac{d^2u}{dx^2} + 4u = 2\sin 2x$$

(7+7)

9. a) The Fourier series $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ is not always convergent. Write down the conditions (Dirichlet's) of a convergent Fourier series. If $f(x)$ is discontinuous at $x = c$, what value of the function should be used at the point of discontinuity, for all practical purpose.

b) Calculate the Laplace transform of the periodic function

$$f(t) = \begin{cases} A, 0 < t < a \\ -A, a < t < 2a \end{cases}$$

where A is a constant and $f(t) = f(t + 2a)$.

(6+8)

****