MATHEMATICS

1. 2 marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. The probability that the first drawn marble is red and second is white is:
    a.   0.06333
    b.   0.05333
    c.   0.02433
    d.   0.05666

2. If $L\{f(t)\} = \frac{e^{\frac{-1}{s}}}{s}$, then $L\{e^{-t}f(3t)\}$ is
    a.   s
    b.   s+1
    c.   s-1
    d. s^2

3. The directional derivative of $f(x, y, z) = 4e^{2x-y+z}$ at the point (1, 1, -1) in the direction towards the point (-3, 5, 6) is
    a. $\frac{-20}{9}$
    b. $\frac{20}{9}$
    c. $\frac{9}{20}$
    d. $\frac{-9}{20}$

4. The integral equation $\int_0^\infty f(x)\sin xt\, dx = \begin{cases} 1, & 0 \le t < 1 \\ 2, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$ is

    a. $\frac{2}{\pi x}$ (1+cos x-2 cos2x)

    b. $\frac{2}{\pi x}$ (1-cos x+2 cos2x)

    c. $\frac{2}{\pi x}$ (cos x-2 cos2x)

    d. $\frac{2}{\pi x}$ (cos x-2 cos2x-1)

5. By dividing [0, 1] into 4 equal sub intervals, the value of $\int_0^1 \frac{dx}{1+x}$ (using trapezoidal rule) correct to 3 decimal places is:
    a.   0.693
    b.   0.694
    c.   0.697
    d.   0.699

6. If u = x²-y², x=2r-3s+4, y=-r+8s-5, then $\frac{\partial u}{\partial r}$ =
   a.   4x+2y
   b.   2x+4y
   c.   4x-2y
   d.   2x-4y

7. The total mass of the region in the cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ with density at any point given by xyz is
   a.   1/8
   b.   2/3
   c.   5/7
   d.   3/2

8. Two circles x²+y²-4x+10y+20 = 0 and x²+y²+8x-6y-24 = 0
   a.   Touch externally
   b.   Touch internally
   c.   Are orthogonal
   d.   Are disjoint

9. If the vectors xi+j-2k, i+j+3k, 8i+5j are coplanar, then the value of x is
   a.   -2
   b.   5
   c.   2
   d.   -5

10. The Laplace transformation of the following function using second translation theorem: 4sin(t-3)u(t-3) is
   a. $e^{3s}\frac{4}{(s^2+1)}$
   b. $e^{3s}\frac{4}{(s^2-1)}$
   c. $e^{-3s}\frac{4}{(s^2+1)}$
   d. $e^{-3s}\frac{4}{(s^2-1)}$

11. The integral of $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\,dy$ is

    a. $\frac{\pi}{2}$

    b. $\frac{\pi}{4}$

    c. $\frac{\pi}{8}$

    d. $\frac{\pi}{6}$

12. The $\int y^2 dx - 2x^2\,dy$ along the parabola $y = x^2$ from $(0, 0)$ to $(2, 4)$ is

    a.   48/5

    b.   -48/5

    c.   8/5

    d.   48

13. If $T: R^3 \rightarrow R^3$ is given by $T(x, y, z) = (x + y + z, y + z, z)$ for $(x, y, z) \in R^3$ then $T^{-1}(x, y, z)$ is

    a. $(x - y, y - z, z - x)$

    b. $(x, y - z, z - x)$

    c. $(x - y, y - z, z)$

    d. $(x + y, y + z, z)$

14. If C is the midpoint of AB and P is a point outside AB, then

    a.   PA+PB=PC

    b.   PA+PB+PC=0

    c.   PA+PB=2PC

    d.   PA+PB+2PC=0

15. If A be a 3 x 3 matrix with Eigen values 1,-1,0 then the determinant of $I+A^{100}$ is

    a.   6

    b.   4

    c.   9

    d.   100

16. The radius of curvature for the curve $y=e^x$ at $(0,1)$ is

    a. $\sqrt{2}$

    b. $2\sqrt{2}$

    c. $\frac{1}{\sqrt{2}}$

    d. $\frac{1}{2\sqrt{2}}$

17. The mean value of a sine wave over half a cycle is
    a.   0.318 x maximum value
    b.   0.707 x maximum value
    c.   Peak value
    d.   0.637 x maximum value

18. Regula Falsi method is used for
    a.   Solution of ordinary differential equation
    b.   Differential of a function
    c.   Integration of a function
    d.   Solution off an algebraic (or) transcendental equation

19. The order of the pole of $\frac{(e^x - 1)}{z^4}$ is
    a.   3
    b.   1
    c.   2
    d.   4

20. If C is unit circle $|z| = 1$ then $\int_c \bar{z} \, dz =$
    a.   0
    b.   1
    c. $2\pi i$
    d. $4\,\pi i$