

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

December 2016 End Semester Examinations
B.Sc. (Nautical Science)- First Semester (2016-17 batch onwards)

Nautical Mathematics (UG21T3102)

Date : 16.12.2016

Time: 3 Hrs

Maximum Marks: 70

Pass Marks : 35

Note: Answer Any Seven Questions. All questions carry equal marks.
Use of approved type scientific calculator is permitted.

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1. a) In a spherical triangle PZY , given $Z = 70^\circ 27'$, $P = 114^\circ 54'$, $Y = 109^\circ 42'$,
Find p and z .
- b) In a spherical triangle PAB , given $a = 57^\circ 00'$, $B = 94^\circ 01'$, $P = 71^\circ 51.5'$,
Find b and p by using the four part formula. (5+5 marks)
2. a) In a spherical triangle ABC , given $A = 100^\circ 50'$, $B = 73^\circ 10'$, $a = 90^\circ$,
Find C , c and b .
- b) In a spherical triangle PAB , given $A = 111^\circ 58'$, $B = 101^\circ 31'$, $P = 90^\circ$,
Find a , p and b . (5+5 marks)
3. a) Prove that $\left[\frac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha} \right]^n = \cos\left(\frac{n\pi}{2} - n\alpha\right) + i \sin\left(\frac{n\pi}{2} - n\alpha\right)$.
- b) Prove that $\sin^5\theta = \frac{1}{10}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$. (5+5 marks)
4. a) Solve the equation $7 \cosh x + 8 \sinh x = 1$ for real values of x .
- b) Prove that $\tan\left[i \log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$. (5+5 marks)
5. a) Find the n^{th} derivative of $e^{5x} \cos x \cos 3x$.
- b) If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. (5+5 marks)
6. a) Using Maclaurin's series, expand the function $\log \sec x$.
- b) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ if $u = \tan^{-1}\left[\frac{2xy}{x^2-y^2}\right]$. (5+5 marks)
7. a) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
- b) Find the maximum and minimum values of $xy + \frac{a^3}{x} + \frac{a^3}{y}$. (5+5 marks)

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.

b) Evaluate $\iint xy(x+y) dx dy$ over the area between x^2 and $y = x$.

(5+5 marks)

9. a) Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dx dy dz$.

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(5+5 marks)

10. a) Show that $\int_0^\infty \frac{x^4}{4^x} dx = \frac{\Gamma 5}{(\log 4)^5}$.

b) Prove that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \beta(m, m)$

(5+5 marks)
