

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

June 2016 End Semester Examinations
B.Sc. (Nautical Science) - 2013 batch onwards
Semester I
Applied Mathematics – I
(UG21T2104)

Date : 16.06.2016/A.N

Time: 3 Hrs

Maximum Marks: 70

Pass Marks : 35

NOTE: Attempt any SEVEN questions. All questions carry equal marks **7x10=70**
Use of Non-programmable scientific calculator is allowed.

1. a. Reduce $1 - \cos \alpha + i \sin \alpha$ to the modules, amplitude form.
b. Find the real values of x, y so that $-3 + ix^2y$ and $x^2 + y + 4i$ may represent complex conjugate numbers
2. a. The centre of a regular hexagon is at the origin and one vertex is given by $\sqrt{3} + i$ on the argand diagram. Determine the other vertices.
b. Find the locus of Z when,
(i) When $\frac{Z+i}{Z+2}$ is real. (ii) When $\frac{Z-i}{Z-2}$ is purely imaginary.
3. Find what curve $Z\bar{Z} + (1+i)Z + (1-i)\bar{Z} = 0$ represents?
4. a. If α, β are the roots of the equation, $x^2 - 2x + 4 = 0$ prove that
$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$$

b. Expand $\cos^7 \theta$ in terms of $\cos \theta$
5. a. Find the n^{th} derivative of $e^{5x} \cos x \cos 3x$
b. If $y = \log (x + \sqrt{1+x^2})^2$ prove that
$$(1+x^2) y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$$
6. a. Verify Rolle's theorem for $f(x) = (x+2)^3 (x-3)^4$ in $(-2, 3)$.

b. Verify Lagrange's mean value theorem for the function $f(x) = \log_e x$ in $[1, e]$

7. Expand $e^{\sin x}$ by Maclaurin's series upto to the term containing x^4

8. a. If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

b. If $\sin u = \frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}$ show that $xu_x + yu_y + zu_z = 0$

9. (a) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$

(b) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 sq. cm.