

INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

June 2016 End Semester Examinations
B.Sc. (Nautical Science) - 2013 batch onwards
Semester III
Applied Mathematics – III
(UG21T2302)

Date : 05.07.2016/F.N

Time: 3 Hrs

Maximum Marks: 70

Pass Marks : 28

NOTE: Attempt any SEVEN questions. All questions carry equal marks
Use of Non-programmable scientific calculator is allowed.

7x10=70

1. Find the Laplace transform of the function

a. $f(t) = |t - 1| + |t + 1|, t \geq 0$

b. Find the Laplace transform of $f(t) = \frac{\cos at - \cos bt}{t} + t \sin at$

2. a. Evaluate $L \left\{ t \int_0^t \frac{e^{-t} \sin t}{t} dt \right\}$

b.

Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$

3. a. Find the inverse Laplace transform of

(i) $\tan^{-1} \left(\frac{2}{s^2} \right)$ (ii) $\cot^{-1} s$

b. Use transform method to solve

$\frac{d^2x}{dt^2} - 2 \frac{dx}{dt} + x = e^t$ with $x = 2, \frac{dx}{dt} = -1$ at $t=0$

4. a. Use convolution theorem to evaluate

$$L^{-1} \left[\frac{1}{(s+a)(s+b)} \right]$$

b. Solve the simultaneous equations

$\frac{dx}{dt} + 5x - 2y = t, \quad \frac{dy}{dt} + 2x + y = 0$ given $x = y = 0$ at $t = 0$

using Laplace transform.

5. a. Find the value $J_{\frac{1}{2}}(x)$.

b. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$

6. Prove that

$$J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right]$$

7. Prove

$$e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$$

8. Prove $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$

9. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre Polynomials.